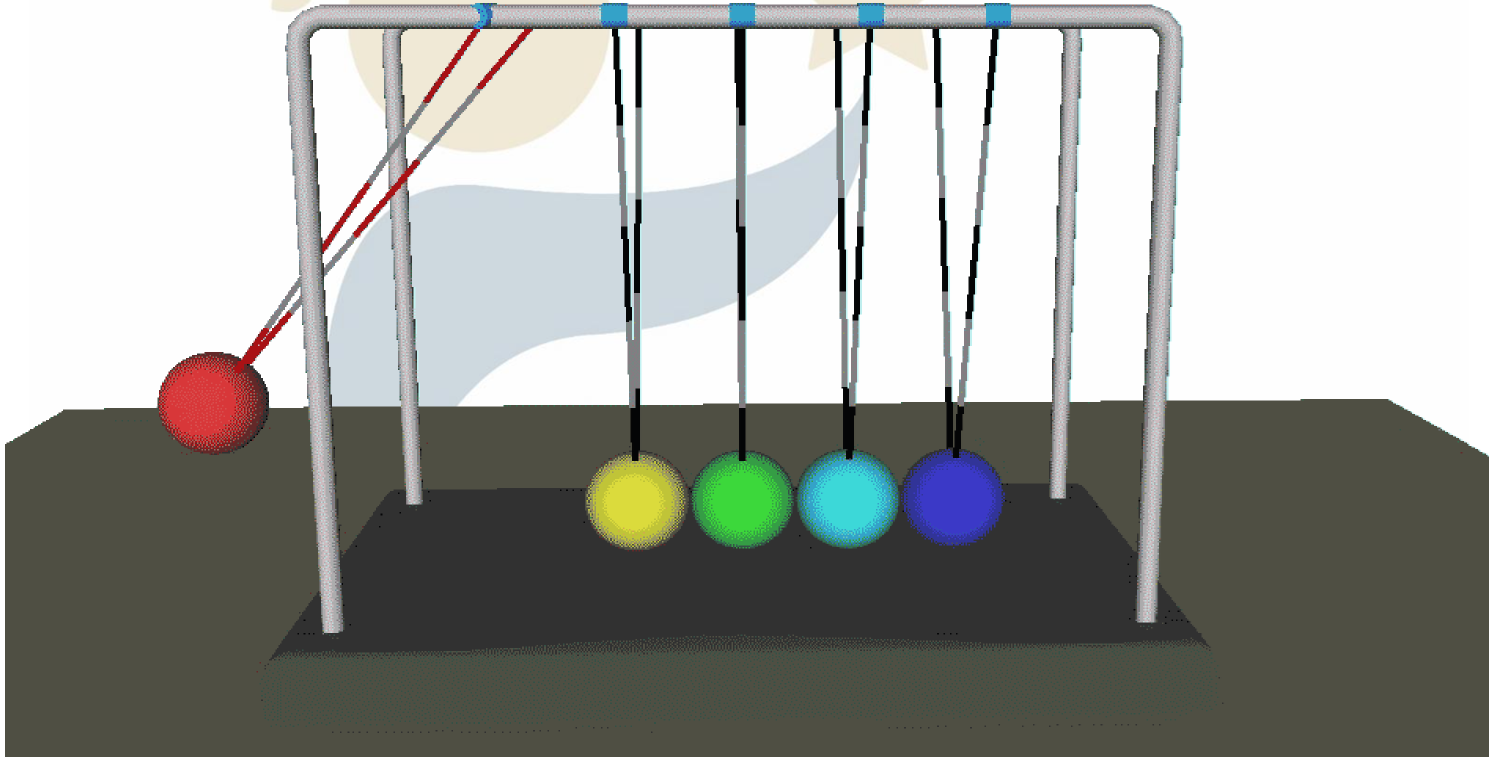
Grade 12 LS – Physics

Chapter 2: Linear Momentum



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Think then Solve

Exercise 1 collision between two objects

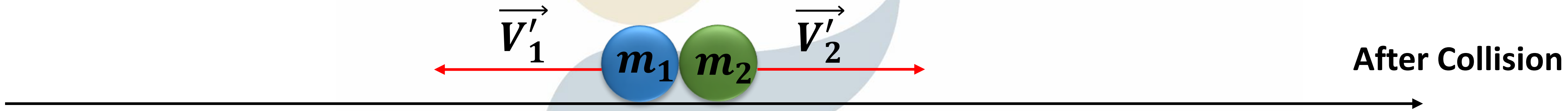
Consider two objects A of mass $m_1 = 500g$ and B of mass $m_2 = 750g$.

A moves horizontally with a velocity magnitude $V_1 = 4m/s$, while B moves with a velocity of magnitude $V_2 = 2m/s$.

At a certain time, a collision takes place between the two objects as shown in the figure.

Exercise 1 collision between two objects

After collision A returns with a velocity $V_1' = 5m/s$ and B moves with a velocity $V_2' = 4m/s$.



1. Determine and represent the forces acting on each object.
2. Determine the sum of external forces acting on the system [A, B]. What can you deduce.

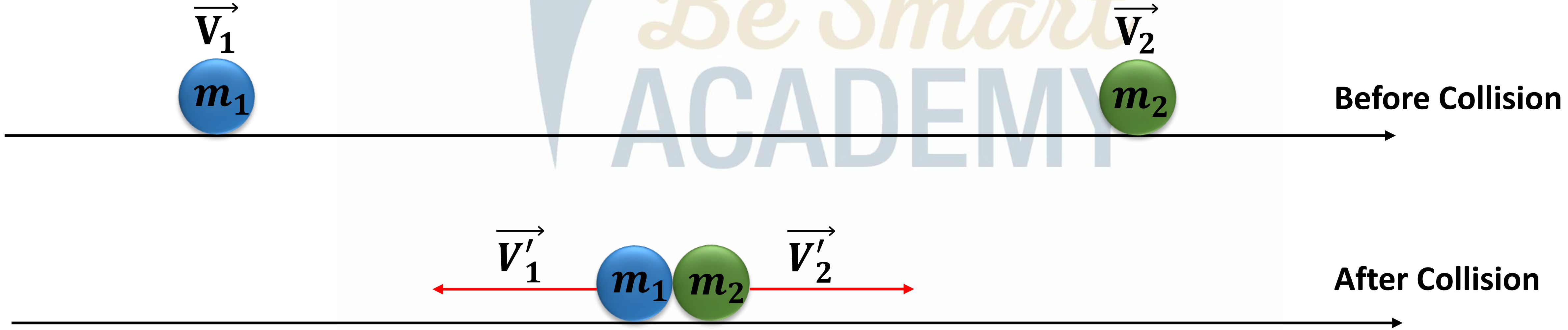
Exercise 1 collision between two objects

3. Calculate the linear momentum $\vec{P}_A$ of object A and $\vec{P}_B$ of object B before collision.

4. Calculate the linear momentum $\vec{P}'_A$ of object A and $\vec{P}'_B$ of object B after collision.

5. Deduce the linear momentums $\vec{P}$ and $\vec{P}'$ of the system [(A) and (B)] before and after collision, respectively

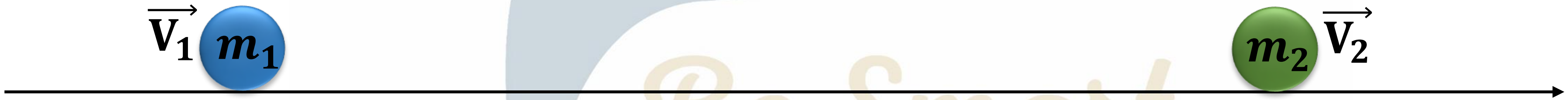
6. Compare $\vec{P}$ and $\vec{P}'$ then conclude



Exercise 1 collision between two objects

$m_1 = 0.5\text{kg};\ V_1 = 4m/s;\ m_2 = 0.75kg;\ V_2 = 2m/s;$
$V'_1 = 5m/s$ and $V'_2 = 4m/s$

1.Determine the forces acting on each object, then the sum of external forces of the system.



For object A: weight ($\overrightarrow{W}_1 = m_1\vec{g}$); normal ($\overrightarrow{N}_1$): $\sum \vec{F}_{ext} = \overrightarrow{W}_1 + \overrightarrow{N}_1 = \vec{0}$.

For object B: weight ($\overrightarrow{W}_2 = m_2\vec{g}$); normal ($\overrightarrow{N}_2$): $\sum \vec{F}_{ext} = \overrightarrow{W}_2 + \overrightarrow{N}_2 = \vec{0}$

Exercise 1 collision between two objects

$m_1 = 0.5\text{kg};\ V_1 = 4m/s;\ m_2 = 0.75kg;\ V_2 = 2m/s;$
$V_1' = 5m/s$ and $V_2' = 4m/s$

2.Determine the sum of external forces acting on the system [A, B]. What can you deduce.

$$\sum \vec{F}_{ext} = \vec{0}. \Rightarrow \frac{\Delta \vec{P}}{\Delta t} = \sum \vec{F}_{ext} = \vec{0}. \Rightarrow \frac{\vec{P}_f - \vec{P}_i}{\Delta t} = \vec{0}.$$

$$\vec{P}_f - \vec{P}_i = \vec{0} \Rightarrow \vec{P}_f = \vec{P}_i$$

The linear momentum of the system is conserved

Exercise 1 collision between two objects

$m_1 = 0.5\text{kg}$; $V_1 = 4m/s$; $m_2 = 0.75kg$; $V_2 = 2m/s$; $V_1' = 5m/s$ and $V_2' = 4m/s$

3. Calculate the linear momentum $\vec{P}_A$ of object A and $\vec{P}_B$ of object B before collision

For (A) before collision: $\vec{P}_A = m_1 \times \vec{V}_1 = 0.5 \times (+4\vec{i})$

$$\vec{P}_A = 2\vec{i} \ (Kg.m / s)$$

For (B) before collision: $\vec{P}_B = m_2 \times \vec{V}_2 = 0.75 \times (-2\vec{i})$

$$\vec{P}_B = -1.5\vec{i} \ (Kg.m/s)$$

Exercise 1 collision between two objects

$m_1 = 0.5\text{kg}; V_1 = 4m/s; m_2 = 0.75kg; V_2 = 2m/s; V'_1 = 5m/s$ and $V'_2 = 4m/s$

4.Calculate the linear momentum $\vec{P}'_A$ of object A and $\vec{P}'_B$ of object B after collision

For (A)after collision:

$$\vec{P}'_A = m_1 \times \vec{V}'_1 = 0.5 \times (-5\vec{i})$$

$$\vec{P}'_A = -2.5\vec{i}\ \text{Kg.m/s.}$$

For (B) after collision:

$$\vec{P}'_B = m_2 \times \vec{V}'_2 = 0.75 \times (4\vec{i})$$

$$\vec{P}'_B = 3\vec{i}\ \text{Kg.m/s.}$$

Exercise 1 collision between two objects

$m_1 = 0.5\text{kg};\ V_1 = 4m/s;\ m_2 = 0.75kg;\ V_2 = 2m/s;$
$\text{V}_1' = 5m/s$ and $\text{V}_2' = 4m/s$

5. Deduce the linear momentums $\vec{P}$ and $\vec{P}'$ of the system [(A) and (B)] before and after collision, respectively.

For the system before collision:

$$\vec{P} = \vec{P}_\text{A} + \vec{P}_\text{B} = 2\vec{\imath} - 1.5\vec{\imath}$$

$$\vec{P} = 0.5\vec{\imath}\ \text{Kg.m/s.}$$

For the system after collision:

$$\vec{P}' = \vec{P}_\text{A}' + \vec{P}_\text{B}' = -2.5\vec{\imath} + 3\vec{\imath}$$

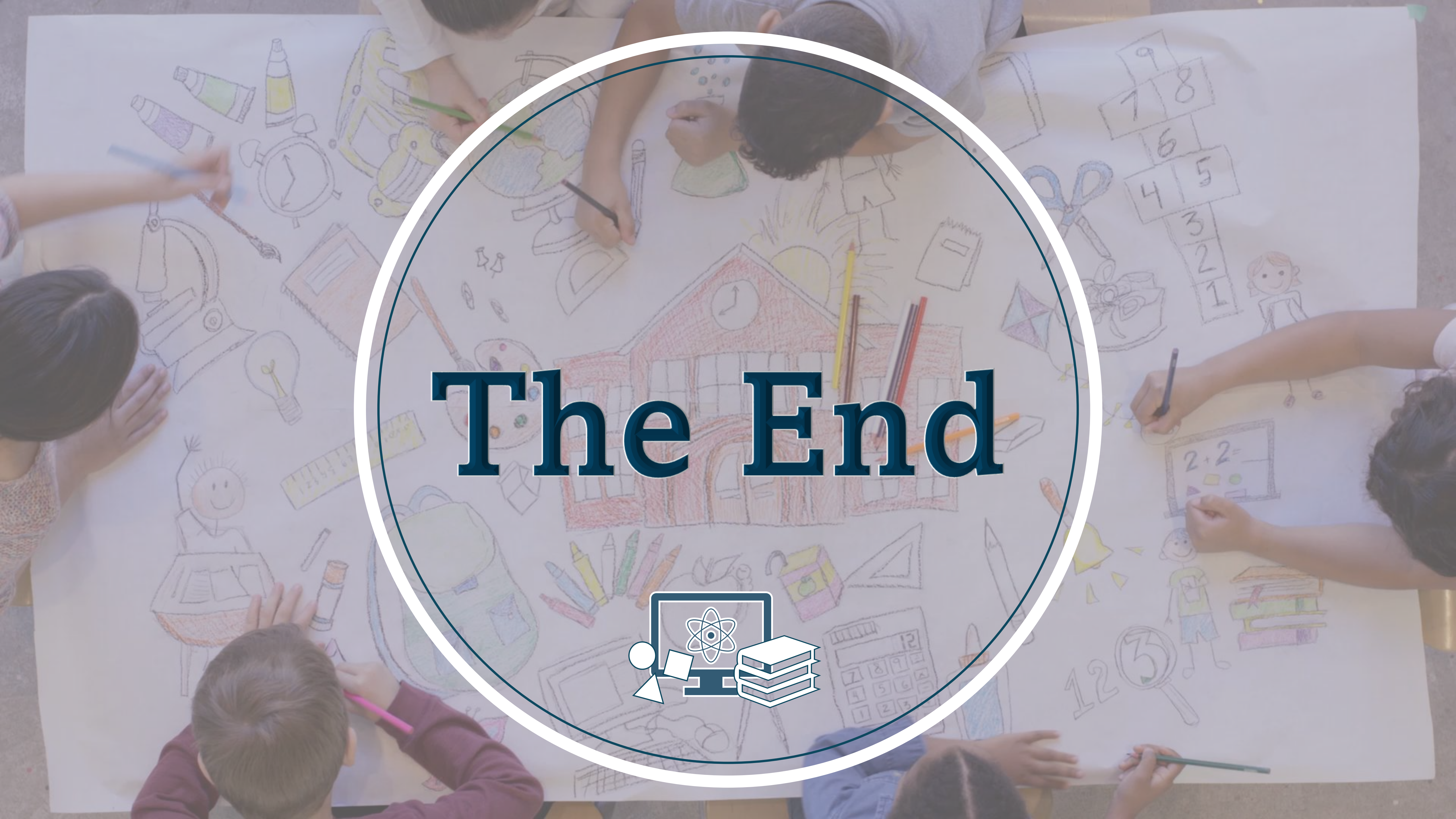
$$\vec{P}' = 0.5\vec{\imath}\ \text{Kg.m/s}$$

Exercise 1 collision between two objects

6. Compare $\vec{P}$ and $\vec{P}'$ then conclude

$$\vec{P} = \vec{P}' = 0.5\vec{i}\ \text{Kg.m/s}$$

The linear momentum is conserved


The End

Think then Solve

Exercise 2 Nature of collision

An object (A), of mass $m_A = 2$ kg, can slide without friction on a path situated in a vertical plane and formed of two parts: a circular part DN and a horizontal rectilinear part NM.

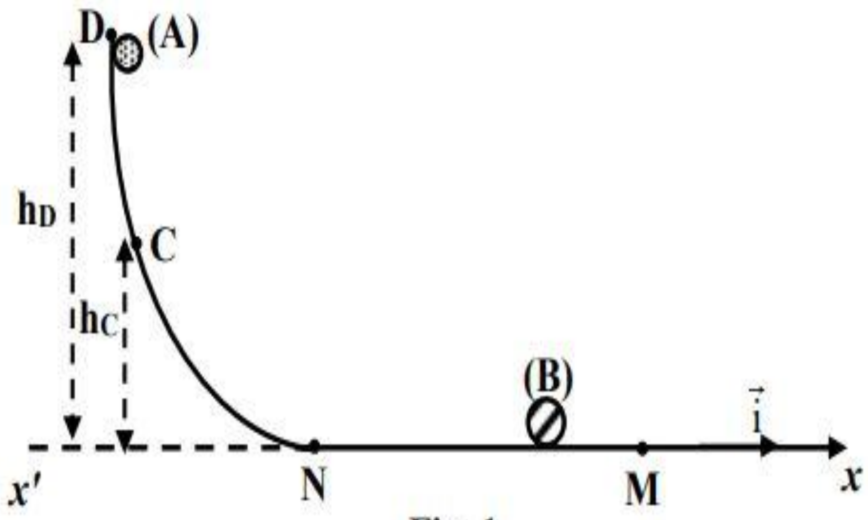


Fig. 1

A is released, without initial velocity, from the point D situated at a height $h_D = 0.45m$ above the horizontal part NM (Fig.1).
The horizontal plane passing through MN is taken as the reference level of gravitational potential energy. Take $g = 10 m/s^2$.

Exercise 2 Nature of collision

1. Calculate the mechanical energy of the system [(A), Earth] at the point D.
2. Deduce the speed V_{1A} of (A) when it reaches the point N.

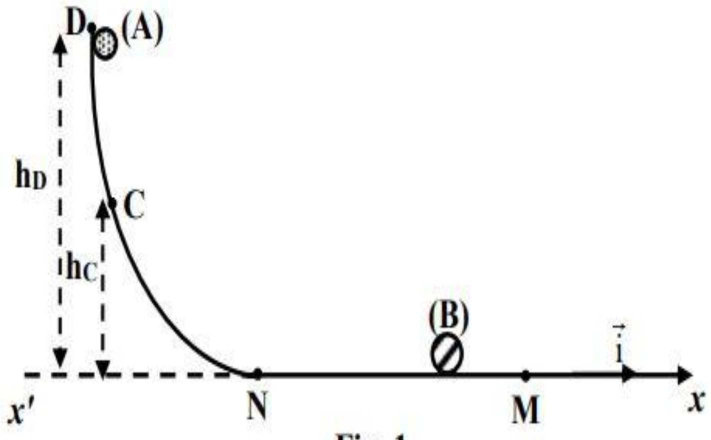


Fig. 1

3. (A) reaches N and moves along NM with the same velocity $\vec{V}_{1A} = V_{1A}.\vec{i}$. Another object (B), of mass $m_B = 4$kg moves from M toward N with the velocity $\vec{V}_{1B} = -1.\vec{i}$.
   a. Determine the linear momentum $\vec{P}_S$ of the system [(A), (B)] before collision.
   b. Deduce the velocity $\vec{V}_G$ of the center of inertia G of the system [(A), (B)].

Exercise 2 Nature of collision

$m_A = 2$ kg; $f = 0$; $V_D = 0$, $h_D = 0.45m$; $g = 10m/s^2$

1. Calculate the mechanical energy of the system [(A), Earth] at the point D.

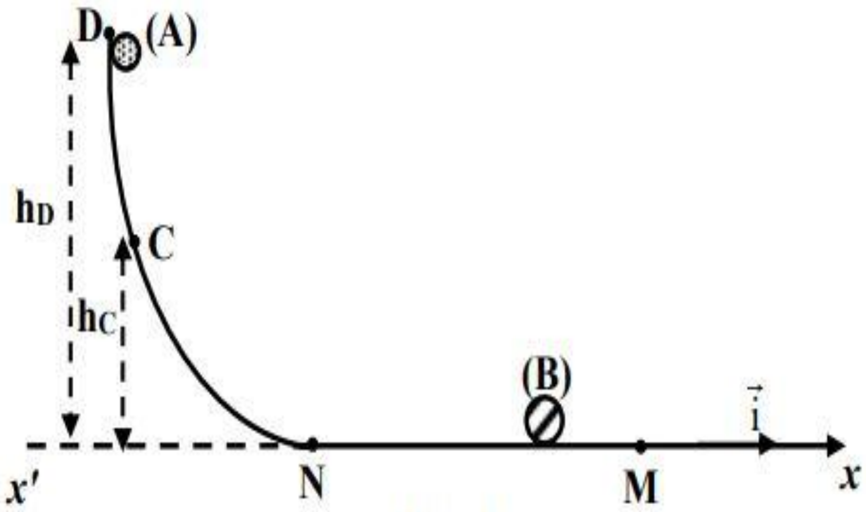


Fig. 1

$$ME_D = KE_D + PE_D$$

$$ME_D = 0 + m_A . g . h_D$$

$$ME_D = 2 \times 10 \times 0.45$$

$$ME_D = 9J$$

Exercise 2 Nature of collision

$m_A = 2$ kg; $f = 0$; $V_D = 0$, $h_D = 0.45m$; $g = 10m/s^2$

2. Deduce the speed V_{1A} of (A) when it reaches the point N.

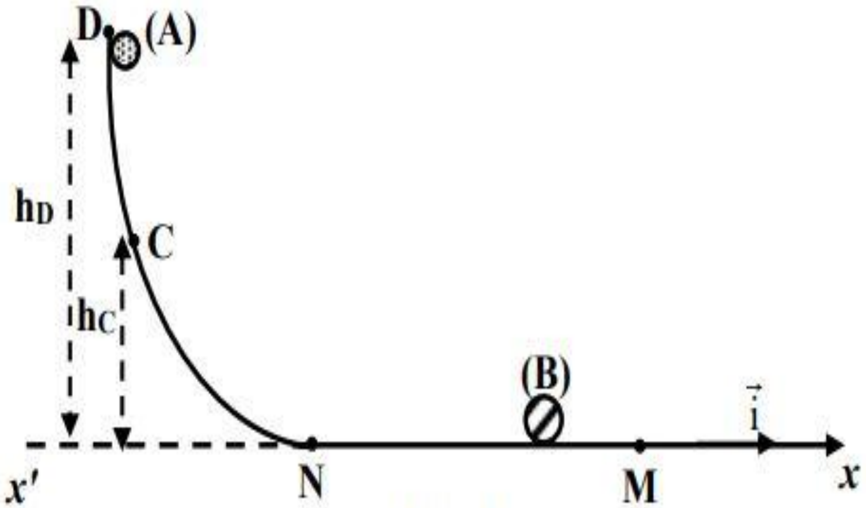


Fig. 1

ME is conserved, because friction in neglected ($f_r = 0$):

$$ME_D = ME_N$$

$$9J = KE_N + PE_N$$

$$9J = \frac{1}{2} m_A V_{1A}^2 + 0$$

$$9J = 0.5 \times 2 \times V_{1A}^2$$

$$V_{1A}^2 = 9J$$

$$V_{1A} = 3m/s$$

Exercise 2 Nature of collision

$m_A = 2\ kg;\ f = 0;\ V_D = 0,\ h_D = 0.45m;\ g = 10m/s^2$

3. (A) reaches N and moves along NM with the same velocity $\vec{V}_{1A} = V_{1A}.\vec{i}$. Another object (B), of mass $m_B = 4$kg moves from M toward N with the velocity $\vec{V}_{1B} = -1.\vec{i}$.

a. Determine the linear momentum $\vec{P}_S$ of the system [(A), (B)] before collision.

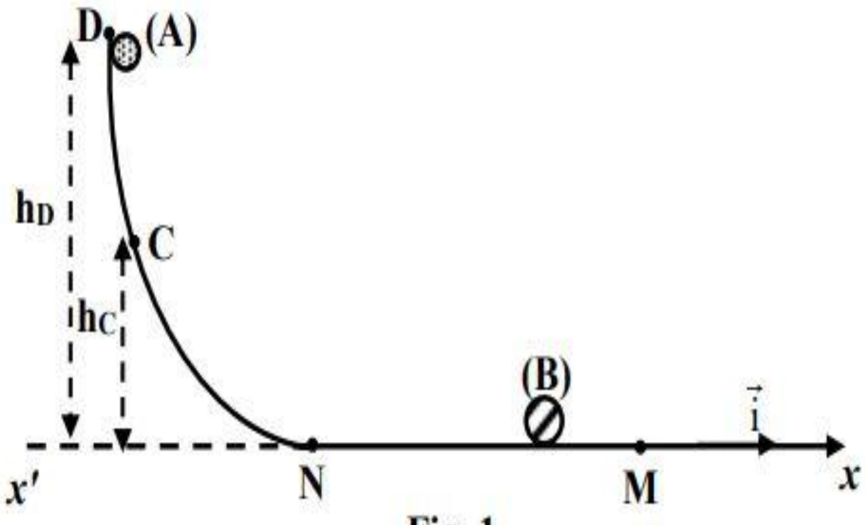


Fig. 1

$$\vec{P}_S = \vec{P}_A + \vec{P}_B$$

$$\vec{P}_S = m_A\vec{V}_A + m_B\vec{V}_B$$

$$\vec{P}_S = 2 \times (+3\vec{i}) + 4 \times (-1\vec{i})$$

$$\vec{P}_S = 2\vec{i} kg.m/s$$

Exercise 2 Nature of collision

$m_A = 2$ kg; $f = 0$; $V_D = 0$, $h_D = 0.45m$; $g = 10m/s^2$

b. Deduce the velocity $\vec{V}_G$ of the center of inertia G of the system [(A), (B)]

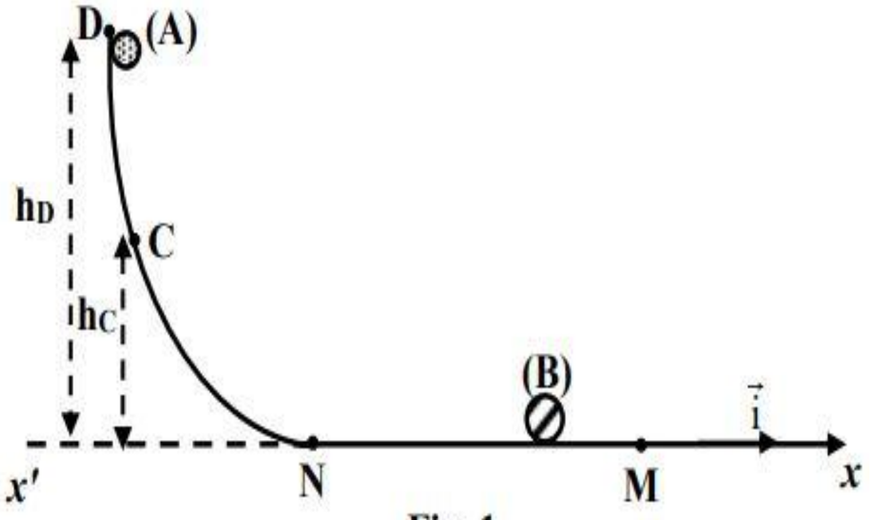


Fig. 1

$$\vec{P}_S = \vec{P}_G = 2\vec{i}kg.m/s$$

$$M \times \vec{V}_G = 2\vec{i}kg.m/s$$

$$(2+4) \times \vec{V}_G = 2\vec{i}kg.m/s$$

$$\vec{V}_G = \frac{2\vec{i}kg.m/s}{6kg}$$

$$\vec{V}_G = 0.33\vec{i}m/s$$

Exercise 2 Nature of collision

4. After collision, (A) rebounds and attains a maximum height $h_C = 0.27m$.
   a. Determine the mechanical energy of the system [(A), Earth] at the point C.
   b. Deduce the speed V_{2A} of (A) just after collision
5. Determine, by applying the principle of the conservation of the linear momentum of the system [(A), (B)], the velocity $\vec{V}_{2B}$ of (B) just after collision.
6. Specify the nature of the collision.

Exercise 2 Nature of collision

4. After collision, (A) rebounds and attains a maximum height $h_C = 0.27m$.

a. Determine the mechanical energy of the system [(A), Earth] at the point C.

$$ME_C = KE_C + PE_C$$

$$ME_C = 0 + m_A g.h_C$$

$$ME_C = 2 \times 10 \times 0.27$$

$$ME_C = 5.4J$$

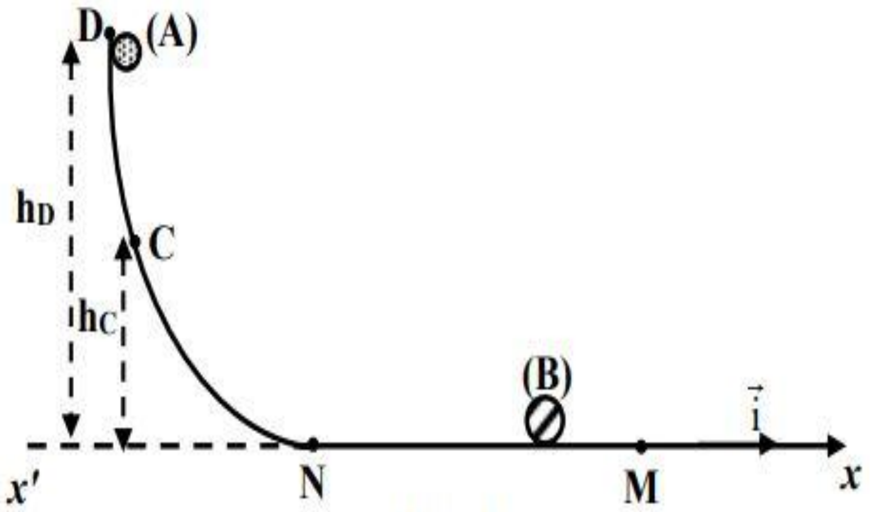


Fig. 1

Exercise 2 — Nature of collision

b. Deduce the speed V_{2A} of (A) just after collision.

ME is conserved; because friction is neglected ($f_r = 0$)

$$ME_C = ME_{after\ collision}$$

$$5.4J = KE + PE$$

$$5.4J = \frac{1}{2} m_A V_{2A}^2 + 0$$

$$5.4J = 0.5 \times 2 \times V_{2A}^2$$

$$5.4J = V_{2A}^2$$

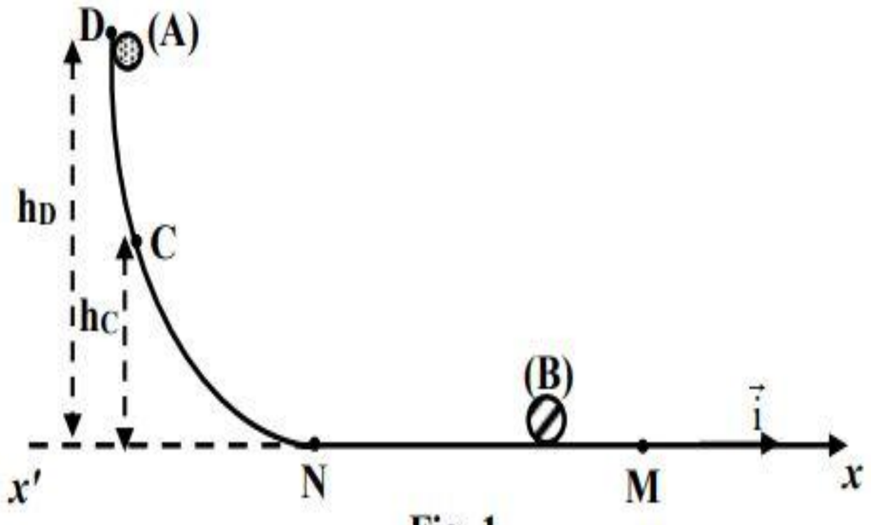


Fig. 1

$$V_{2A} = 2.32 m/s$$

Exercise 2 Nature of collision

5. **Determine, by applying the principle of the conservation of the linear momentum of the system [(A), (B)], the velocity $\vec{V}_{2B}$ of (B) just after collision.**

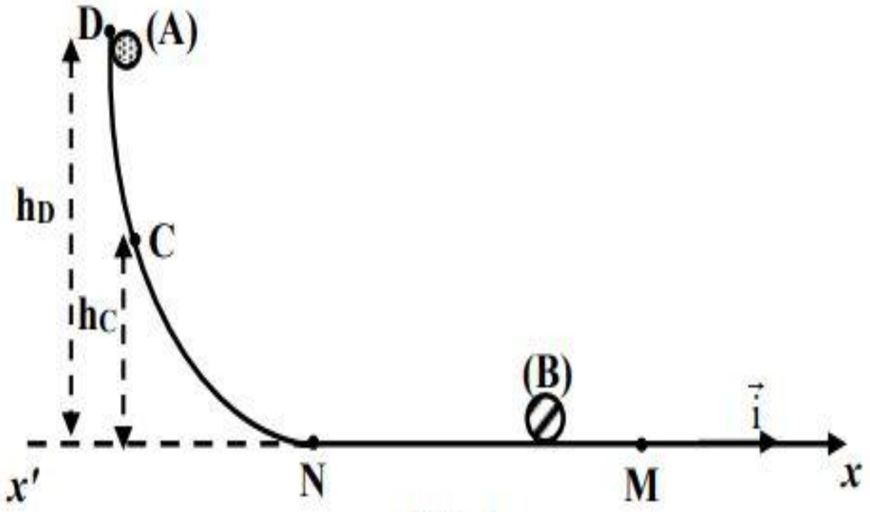


Fig. 1

$$\vec{P}_{s\,(before)} = \vec{P}_{s\,(after)}$$

$$2\vec{i}kg.m/s = \vec{P}_A + \vec{P}_B$$

$$2\vec{i}kg.m\,/\,s = m_A.\vec{V}_{2A} + m_B.\vec{V}_{2B}$$

$$2\vec{i} = 2 \times (-2.32\vec{i}) + 4 \times \vec{V}_{2B}$$

$$2\vec{i} = -4.64\vec{i} + 4 \times \vec{V}_{2B}$$

$$6.64\vec{i} = 4 \times \vec{V}_{2B}$$

$$\boxed{\vec{V}_{2B} = 1.66\vec{i}m/s}$$

6. Specify the nature of the collision.

$$KE_{before} = KE_A + KE_B$$

$$KE_{before} = \frac{1}{2} m_A V_{1A}^2 + \frac{1}{2} m_B V_{1B}^2$$

$$KE_{before} = 0.5 \times 2 \times 3^2 + 0.5 \times 4 \times (-1)^2$$

$$\mathrm{KE_{before}} = 9 + 2 = 11\mathrm{J}$$

Exercise 2 — Nature of collision

$$KE_{after} = KE_A + KE_B$$

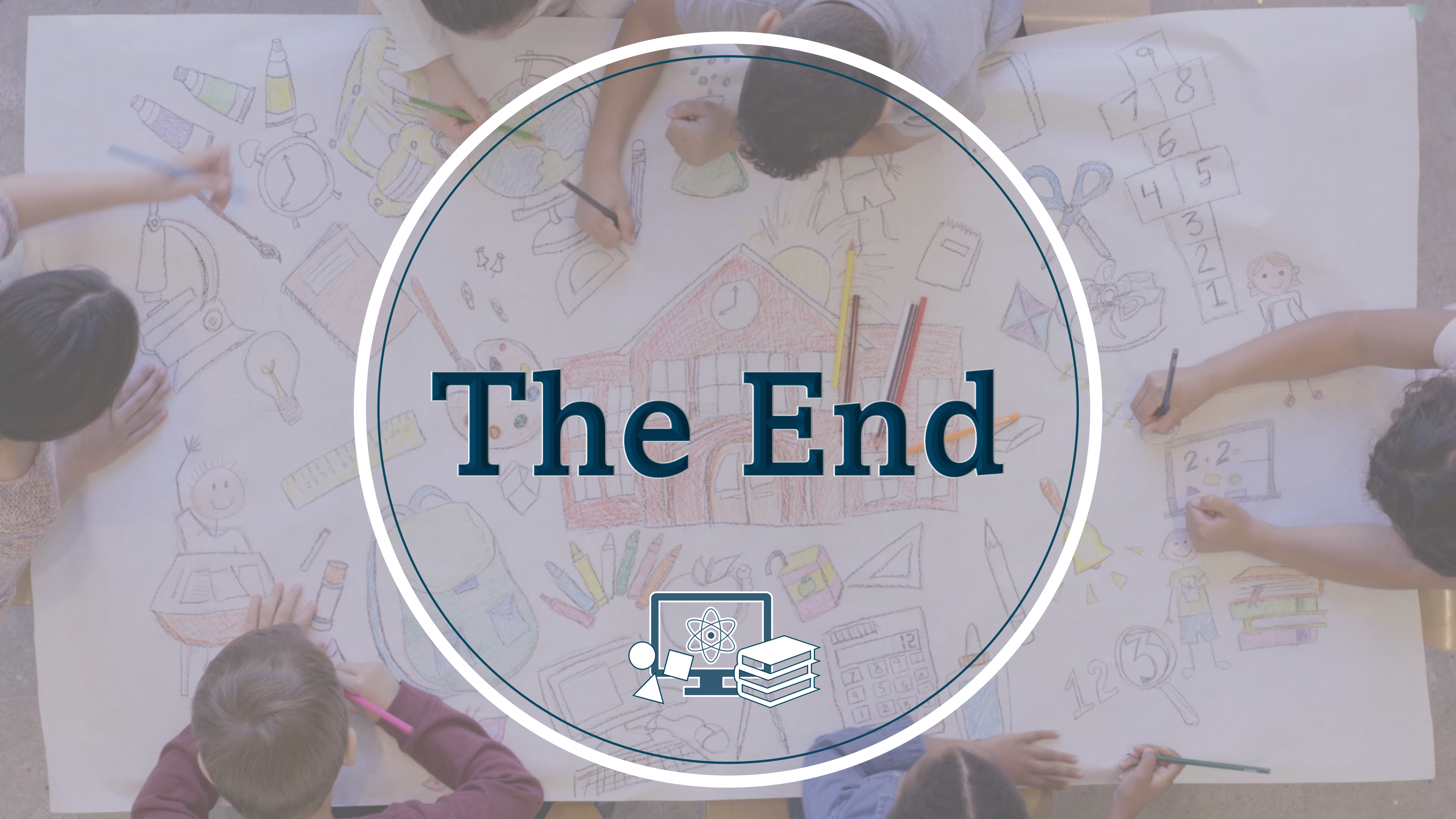
$$KE_{after} = \frac{1}{2} m_A V_{2A}^2 + \frac{1}{2} m_B V_{2B}^2$$

$$KE_{after} = 0.5 \times 2 \times 2.32^2 + 0.5 \times 4 \times (1.66)^2$$

$$KE_{after} = 5.382 + 5.511$$

$$\mathrm{KE}_{after} = 10.95\mathrm{J}$$

$\mathrm{KE_{before}} \approx \mathrm{KE}_{after}$ Then the collision is elastic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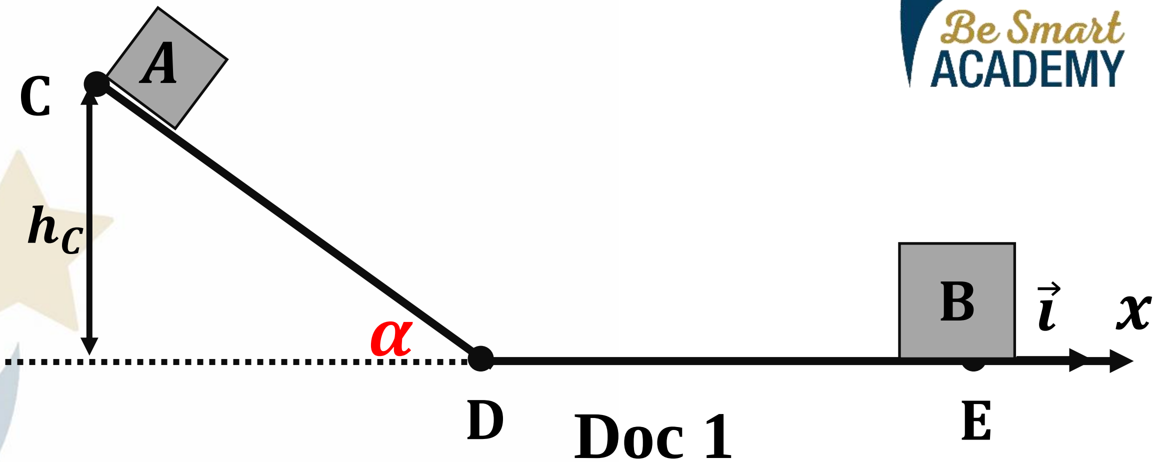
Think then Solve

Exercise 3 Principle of Interaction

The aim of this exercise is to verify the principle of interaction between two blocks.



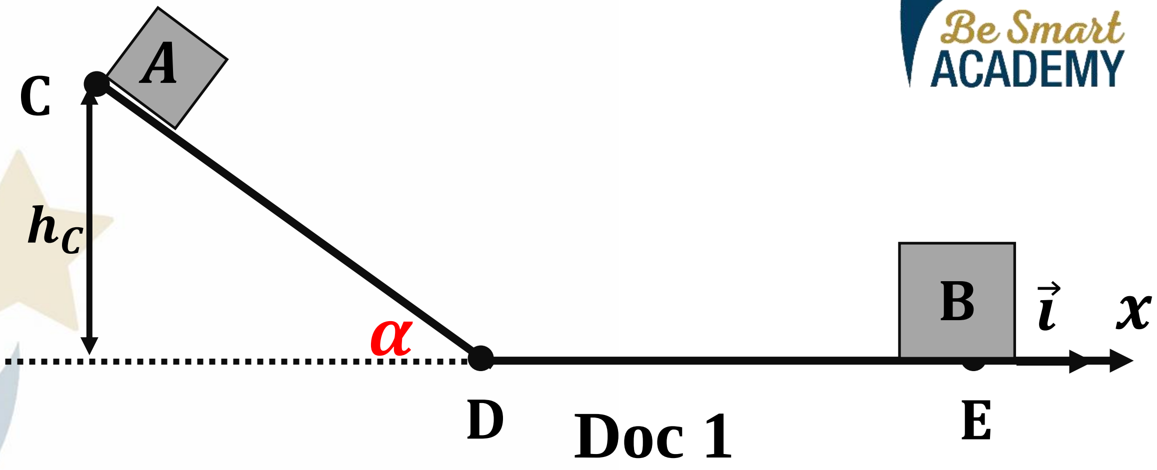
Doc 1

For this purpose, we consider two blocks (A) and (B) considered as particles of respective masses $m_A = 200\text{g}$ and $m_B = 800\text{g}$.

(A) and (B) can move without friction on a track CDE lying in a vertical plane. This track is formed of two parts: the first one CD is straight and inclined by an angle α with respect to the horizontal and the second one DE is straight and horizontal.

Exercise 3 Principle of Interaction

Block (A) is released without initial velocity from point C situated at a height $h_C = 0.2m$ above a horizontal x-axis, confounded with DE, of unit vector



- The horizontal plane containing the x-axis as a reference level for gravitational potential energy;
- $g = 10m/s^2$

Exercise 3 Principle of Interaction

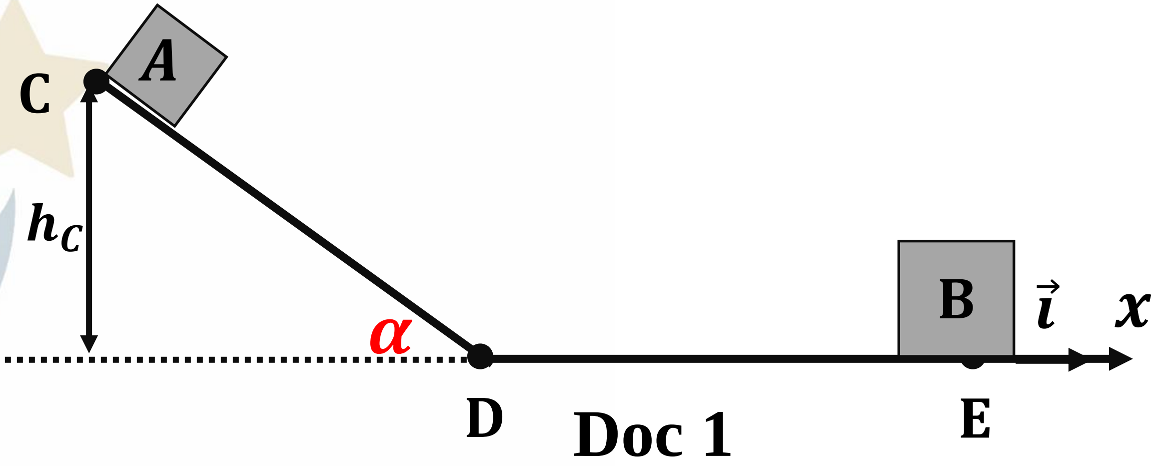
1) The mechanical energy of the system [(A), Track, Earth] is conserved between C and D. Why?

2) Deduce that the speed of (A) at point D is $V_A = 2m/s$.

3) (A) continues its motion with a velocity $\vec{V}_A = 2\vec{i}$ (m/s) along track DE until it makes a head-on elastic collision with (B) initially at rest. Show that the velocities of (A) and (B) right after the collision are $\vec{V}'_A = -1.2\vec{i}$ (m/s) and $\vec{V}'_B = 0.8\vec{i}$ (m/s).

Exercise 3 Principle of Interaction

$m_A = 0.2kg;\ m_B = 0.8kg;\ f = 0N;\ h_C = 0.2m;\ g = 10m/s^2;$
$V_C = 0$



1)The mechanical energy of the system [(A), Track, Earth] is conserved between C and D. Why?

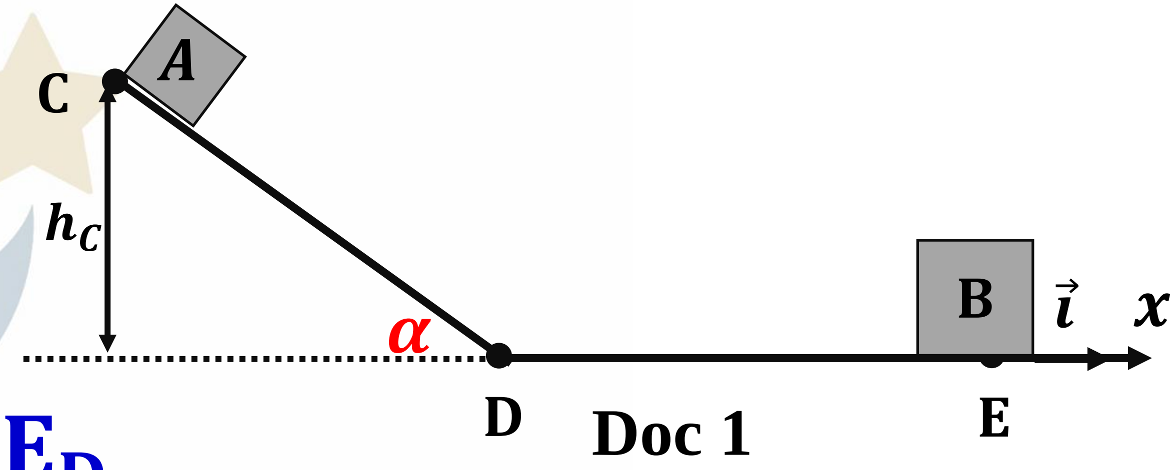
Since the friction is neglected, therefore the mechanical energy is conserved.

0.5 pt

Exercise 3 Principle of Interaction

$m_A = 0.2kg; m_B = 0.8kg; f = 0N; h_C = 0.2m; g = 10m/s^2; V_C = 0$

2)Deduce that the speed of (A) at point D is $V_A = 2m/s$.



ME is conserved , then $ME_C = ME_D$

$$KE_C + GPE_C = KE_D + GPE_D$$

$$0 + mgh_C = \frac{1}{2}mV_A^2 + 0$$

$$10 \times 0.2 = 0.5V_A^2 \Rightarrow 2 = 0.5V_A^2$$

$$V_A^2 = \frac{2}{0.5} \Rightarrow V_A^2 = 4$$

$$V_A = 2m/s$$

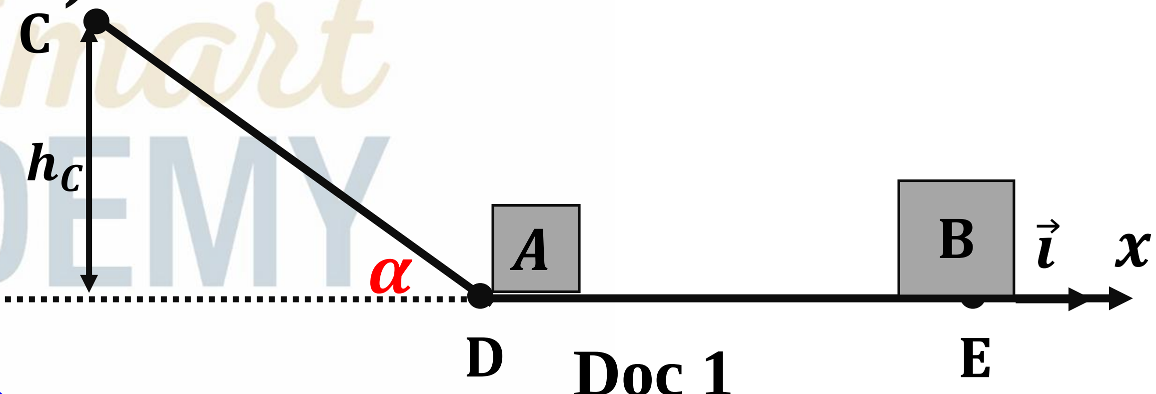
1.5 pt

Exercise 3 Principle of Interaction

$m_A = 0.2\text{kg}; m_B = 0.8\text{kg}; f = 0N; h_C = 0.2m; g = 10m/s^2$

3)(A) continues its motion with a velocity $\vec{V}_A = 2\vec{i}$ (m/s) along track DE until it makes a head-on elastic collision with (B) initially at rest.

Show that the velocities of (A) and (B) right after the collision are $\vec{V}'_A = -1.2\vec{i}$ (m/s) and $\vec{V}'_B = 0.8\vec{i}$ (m/s).



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linear momentum is conserved:

$$\vec{P}_{bef} = \vec{P}_{aft}$$

$$m_A\vec{V}_A + m_B\vec{V}_B = m_A\vec{V}'_A + m_B\vec{V}'_B$$

Exercise 3 Principle of Interaction

$$m_A\vec{V}_A + m_B\vec{V}_B = m_A\vec{V}'_A + m_B\vec{V}'_B$$

$$m_A\vec{V}_A = m_A\vec{V}'_A + m_B\vec{V}'_B$$

The velocities are collinear then:

$$m_AV_A = m_AV'_A + m_BV'_B$$

$$m_AV_A - m_AV'_A = m_BV'_B$$

$$m_A(V_A - V'_A) = m_BV'_B \ldots (1)$$

0.5 pt

Exercise 3 Principle of Interaction

The collision is elastic, then the kinetic energy is conserved:

$$KE_{bef} = KE_{aft}$$

$$\frac{1}{2}m_A V_A^2 + \frac{1}{2}m_B V_B^2 = \frac{1}{2}m_A {V'}_A^2 + \frac{1}{2}m_B {V'}_B^2$$

$$m_A V_A^2 = m_A {V'}_A^2 + m_B {V'}_B^2$$

$$m_A V_A^2 - m_A {V'}_A^2 = m_B {V'}_B^2 \Rightarrow m_A(V_A^2 - {V'}_A^2) = m_B {V'}_B^2$$

$$m_A(V_A - V'_A)(V_A + V'_A) = m_B {V'}_B^2 \ldots\ldots (2)$$

0.5 pt

Exercise 3 Principle of Interaction

Divide (2) ÷ (1) then:

$$\frac{m_A(V_A - V'_A)(V_A + V'_A)}{m_A(V_A - V'_A)} = \frac{m_B V'^2_B}{m_B V'_B}$$

$$V_A + V'_A = V'_B \ldots\ldots\ldots (3)$$

0.5 pt

Exercise 3 Principle of Interaction

$$\begin{cases} m_A(V_A - V'_A) = m_B V'_B \dots (1) \\ V_A + V'_A = V'_B \dots\dots\dots (3) \times m_A \end{cases}$$

$$\begin{cases} m_A V_A - m_A V'_A = m_B V'_B \\ m_A V_A + m_A V'_A = m_A V'_B \end{cases} \dots\dots add$$

$$2m_A V_A = m_B V'_B + m_A V'_B$$

$$2m_A V_A = V'_B(m_B + m_A)$$

$$V'_B = \frac{2m_A V_A}{m_B + m_A}$$

$$V'_B = \frac{2 \times 0.2 \times 2}{0.2 + 0.8}$$

$$V'_B = 0.8 m/s$$

$$\vec{B'_B} = 0.8\vec{i}\ m/s$$

0.25 pt

Exercise 3 Principle of Interaction

To calculate V'_A Substitute $V'_B = 0.8m/s$ in equation 3:

$$V_A + V'_A = V'_B$$

$$2 + V'_A = 0.8$$

$$V'_A = 0.8 - 2$$

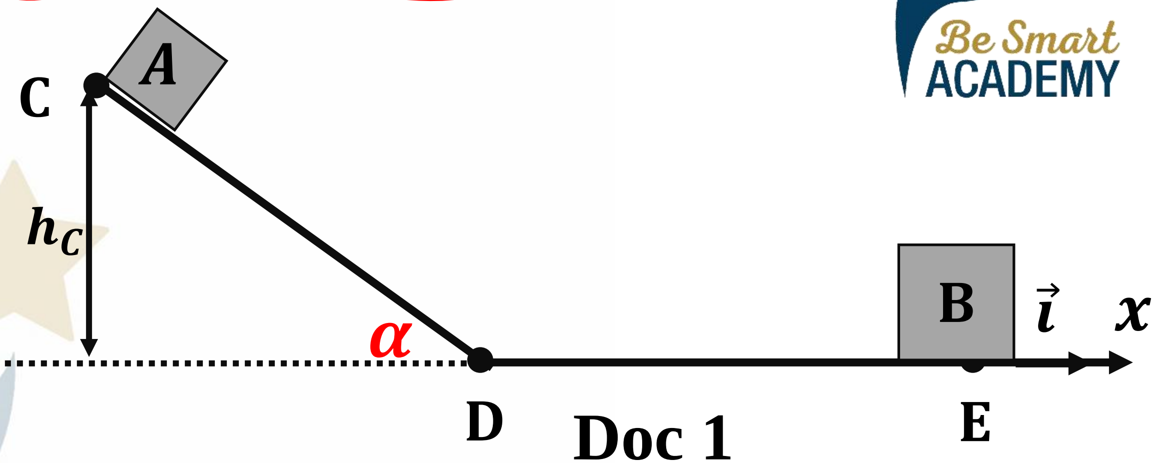
$$V'_A = -1.2m/s$$

$$\vec{B}'_A = -1.2\vec{i}\ m/s$$

0.25 pt

Exercise 3 Principle of Interaction

4)The duration of the collision is $\Delta t = 0.1s$, so $\frac{\Delta\vec{P}}{\Delta t} = \frac{d\vec{P}}{dt}$. Apply, during Newton's second law:



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4.1) on (B) to determine the force $\vec{F}_{A/B}$ exerted by (A) on (B);

4.2) on (A) to determine the force $\vec{F}_{B/A}$ exerted by (B) on (A);

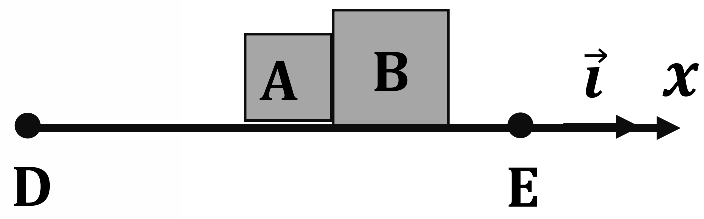
5)Deduce that the principle of interaction is verified.

Exercise 3 Principle of Interaction

$m_A = 0.2\text{kg};\ m_B = 0.8\text{kg};\ f = 0N;\ g = 10m/s^2$

4.1) on (B) to determine the force $\vec{F}_{A/B}$ exerted by (A) on (B);

Apply newton's 2nd law on B:



$$\sum \vec{F}_{ex} = \frac{\Delta \vec{P}}{\Delta t}$$

$$m_B\vec{g} + \vec{N}_B + \vec{F}_{A/B} = \frac{m_B\vec{V}'_B - m_B\vec{V}_B}{\Delta t}$$

Exercise 3 Principle of Interaction

$$m_B\vec{g} + \vec{N}_B + \vec{F}_{A/B} = \frac{m_B\vec{V}'_B - m_B\vec{V}_B}{\Delta t}$$

Project along x-axis: $m_B\vec{g} + \vec{N}_B = \vec{0}$

$$\vec{F}_{A/B} = \frac{m_B\vec{V}'_B - m_B\vec{V}_B}{\Delta t} \Rightarrow \vec{F}_{A/B} = \frac{0.8(0.8\vec{i}) - 0}{0.1}$$

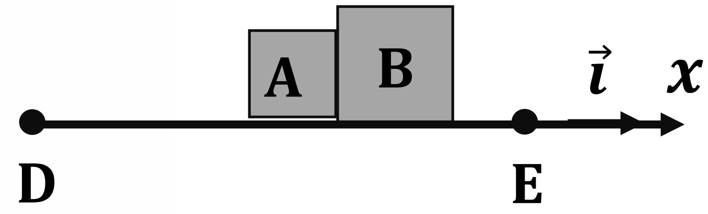
$$\vec{F}_{A/B} = 6.4\vec{i}\ (N)$$

1 pt

Exercise 3 Principle of Interaction

$m_A = 0.2\text{kg};\ m_B = 0.8\text{kg};\ f = 0N;\ g = 10m/s^2$

4.2) on (A) to determine the force $\vec{F}_{B/A}$ exerted by (B) on (A);



Apply newton's 2nd law on A:

$$\sum \vec{F}_{ex} = \frac{\Delta\vec{P}}{\Delta t}$$

$$m_A\vec{g} + \vec{N}_A + \vec{F}_{B/A} = \frac{m_A\vec{V}'_A - m_A\vec{V}_A}{\Delta t}$$

$$m_A\vec{g} + \vec{N}_A + \vec{F}_{B/A} = \frac{m_A\vec{V'}_A - m_A\vec{V}_A}{\Delta t}$$

Project along x-axis: $m_A\vec{g} + \vec{N}_A = \vec{0}$

$$\vec{F}_{B/A} = \frac{m_A\vec{V'}_A - m_A\vec{V}_A}{\Delta t} \Rightarrow \vec{F}_{B/A} = \frac{0.2(-1.2\vec{i}) - 0.2(-2\vec{i})}{0.1}$$

$$\vec{F}_{B/A} = -6.4\vec{i}\ (N)$$

0.5 pt

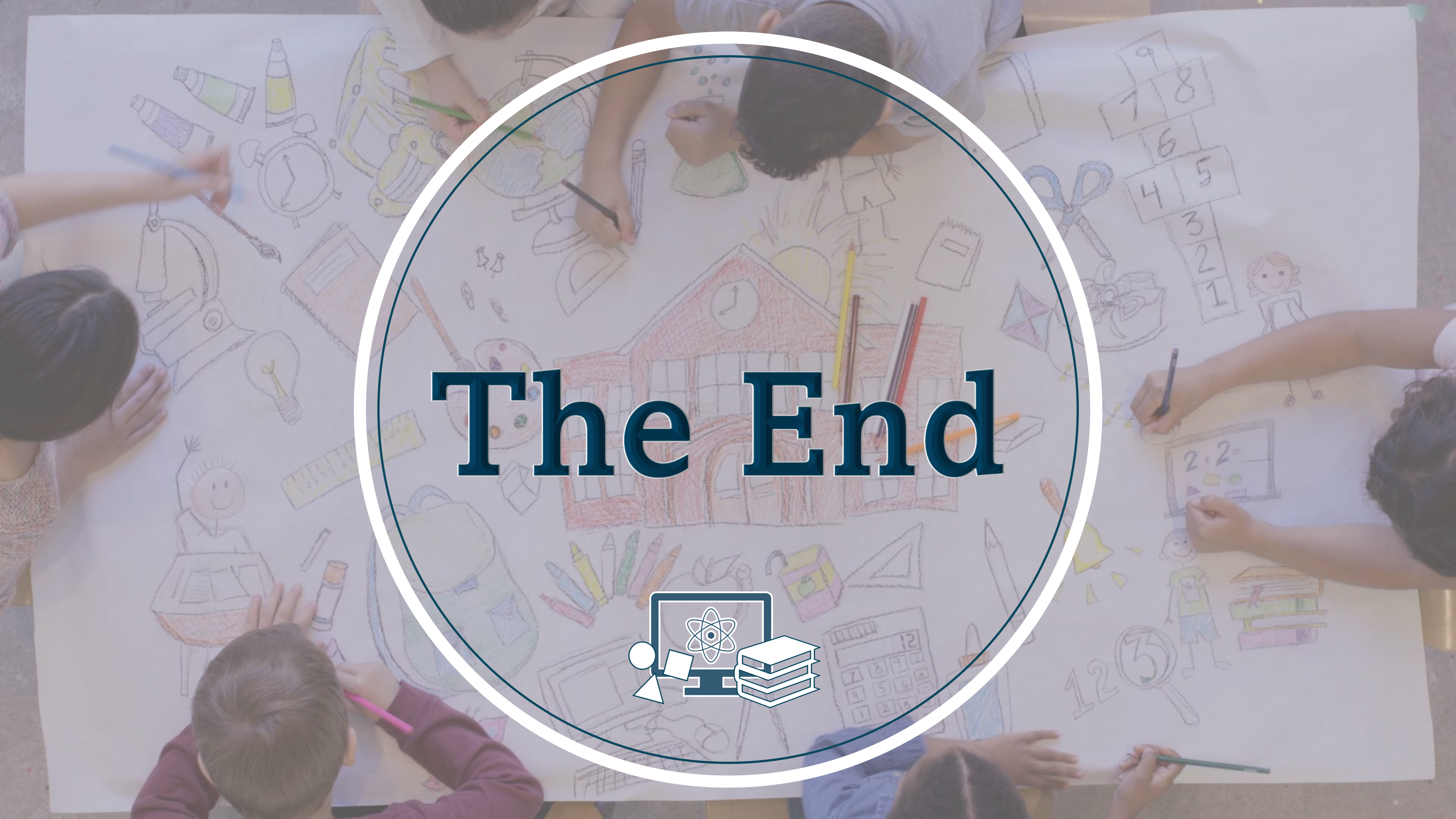
Exercise 3 Principle of Interaction

5) Deduce that the principle of interaction is verified.

$$\vec{F}_{B/A} = -\vec{F}_{A/B}$$

Then the principle of interaction is verified.

0.5 pt


The End

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